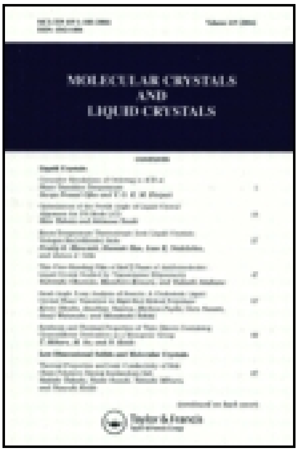

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Investigation of Multiple Scattering in Polymer-Dispersed Liquid Crystal Films on the Base of Radiative Transfer Equation

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Investigation of Multiple Scattering in Polymer-Dispersed Liquid Crystal Films on the Base of Radiative Transfer Equation

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A model of light transfer in polymer dispersed liquid crystal (PDLC) films has been developed. It takes into account the anisotropy of nematic liquid crystal droplets, the droplets polydispersity, multiple scattering, and Fresnel reflection at the film interfaces. The model is based on the solution of the radiative transfer equation (RTE) by the adding method extended for anisotropic liquid crystal droplets.

Keywords: PDLC valves, multiple scattering.

INTRODUCTION

For proper description of light scattering in PDLC films, one has to take into account shape, size, and polydispersity of droplets, nematic director field within a droplet, optical interaction of droplets, multiple scattering of light, Fresnel reflection at the interfaces of the film.

There are a lot of works on the problem of single scattering in PDLC films, but the investigations of multiple scattering in such films are only at the beginning. In this work a method for calculation of angular light distribution in PDLC films with spherical droplets is developed for oblique illumination by unpolarized light. The dependence of the unit volume parameters on the illumination direction, droplet polydispersity, and Fresnel reflection at the film interfaces are taking into account. The method is applied for optically thin films, when multiple scattering is small, and optically thick films, when the contribution of multiple scattering is significant. It allows one to consider a gray scale for such films.

We restricted our investigation by the case of independent scattering and anomalous diffraction approximation for scattering by a separate droplet. The problem of dependent scattering can be solved by a proper change of unit volume parameters. To make the range of considered droplet diameters wider, it is possible to use another approaches for scattering by a separate droplet.[1-6] The base of our consideration is the solution of the radiative transfer equation by the adding method extended by us for PDLC films, where a major feature is the dependence of unit volume parameters on the direction of impinging light. The angular distributions of scattered light and contrast ratios in the transparency mode are presented.

RADIATIVE TRANSFER EQUATION

To describe light intensity in a PDLC film, the radiative transfer theory[7, 8] is used. In the isotropic case the scattering characteristics of a medium in the radiative transfer equation (RTE) are: scattering σ, absorption α, and extinction $\varepsilon = \sigma + \alpha$ coefficients. These coefficients are connected with the scattering Σ_s, absorption Σ_α, and extinction Σ_e cross-sections by the relations: $\sigma = N\Sigma_s$; $\alpha = N\Sigma_\alpha$; $\varepsilon = N\Sigma_e$, N is the number of droplets per unit volume. One has to know too the phase function $X(\cos\gamma)$, which described the angular distribution of light scattered by the unit volume of the medium; γ is the scattering angle (the angle between the direction of the incident and scattered light); $\cos\gamma = \mu\mu' + \sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\varphi-\varphi')$, where $\mu = \cos\theta$ and $\mu' = \cos\theta'$ are the cosines of the axial angles of light scattered and incident on the unit volume; φ and φ' are the azimuthal angles of light scattered and incident on the unit volume, respectively. In the OFF state, when the droplets directors are oriented randomly, these characteristics do not depend on light propagation direction. For such dispersions, reliable methods for the solution of the radiative transfer equation have been developed and the peculiarities of radiative transfer have been well studied.[7-10] In the ON state, when the droplet directors are oriented in the direction of a strong external field, the elementary volume characteristics are anisotropic. They depend on light propagation direction, and the distribution of light in the layer has some peculiarities.

Following to the authors,[2, 11, 12] we consider bipolar droplets, assuming that the director configuration can be modeled as homogeneous. Some results on the opportunity to make this assumption were presented.[13] The more the difference between the ordinary refractive indices of a liquid crystal and a binder, the better is this assumption. The film is illuminated at angle θ_{0e} (Figure 1) from below by an azimuthally symmetrical light beam with intensity I_0. The light is scattered and absorbed by the droplets. A portion of the scattered light passes through the film and leaves it through the upper interface, the rest of the scattered light leaves the film through the lower

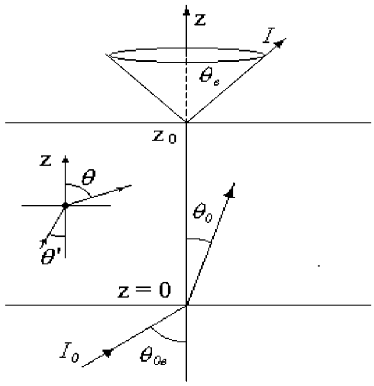

FIGURE 1. Schematic representation of scattering geometry in a film of thickness z_0 (section in the yz plane). Notations are in the text.

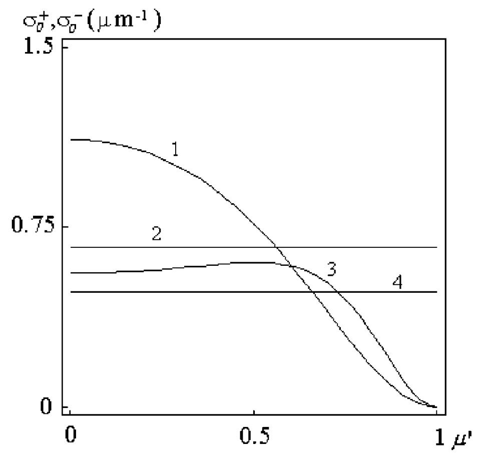

FIGURE 2. Angular dependencies of the scattering coefficients for PDLC films in the ON state (curves 1, 3) and in the OFF state (curves 2, 4) at mean radius $\overline{R}$=0.5 ìm, variation coefficient C_v=0.4 (curves 1, 2) and at $\overline{R}$=1.2 ìm, C_v=0.4 (curves 3, 4), n_e=1.7, $n_0 = n_m$=1.55.

interface. In the case of applied field, the PDLC film is an anisotropic, axial-symmetric system. For such a system the extinction and scattering coefficients depend on the axial angle of light propagation ($\sigma = \sigma(\mu), \varepsilon = \varepsilon(\mu)$), and phase function depends on axial angles and on the difference of incident and scattered azimuthal angles ($X = X(\mu, \mu', \varphi - \varphi')$). The RTE for an anisotropic layer with Fresnel interfaces illuminated by a wide azimuthally symmetrical unpolarized light beam can be derived from the general equation[7] by integration on azimuth and taking into account directed light propagating upward and downward in the film:

$$\mu \frac{\partial I(z,\mu)}{\partial z} + \varepsilon(\mu) I(z,\mu) = \int_{-1}^{1} X(\mu,\mu')\sigma(\mu')I(z,\mu')d\mu' + \\ + X(\mu,\mu_0)\sigma(\mu_0)I_0^{\uparrow} e^{-\frac{\varepsilon(\mu_0)z}{\mu_0}} + X(-\mu,\mu_0)\sigma(\mu_0)I_0^{\downarrow} e^{-\frac{\varepsilon(\mu_0)(z_0-z)}{\mu_0}}. \quad (1)$$

Here $\mu_0 = \cos\theta_0$ is the cosine of the angle at which a parallel light beam propagates in the film; z is the film depth; z_0 is the film thickness; $I(z,\mu)$ is the azimuth-averaged intensity of light propagating at depth z at angle $\arccos\mu$; $\varepsilon(\mu)$ and $\sigma(\mu)$ are the extinction and scattering coefficients depending on the impinging light; $X(\mu,\mu') = \frac{1}{2\pi}\int_0^{2\pi} X(\mu,\mu',\varphi-\varphi')d\varphi)$ is the redistribution function.[10] $I_0^{\uparrow}$ and $I_0^{\downarrow}$ are the intensities of collimated light beams, propagating upward and downward at the lower and upper interfaces inside the film.

Normally, the refractive indices of the polymer matrix and glass plates bounded the matrixare close in value. Therefore, we assume that no reflection occurs at the matrix-glass interface and restrict ourselves to the account of the Fresnel reflection at the air-glass interface:

$$I(\tau = 0, \mu) = \kappa(\mu) I(\tau = 0, -\mu),$$
$$I(\tau = \tau_0, -\mu) = \kappa(\mu) I(\tau = \tau_0, \mu). \quad (2)$$

Here $\kappa(\mu)$ stands for the reflection coefficient at the film interface;[14] the optical depth $\tau = \varepsilon_m z$ and the optical thickness $\tau_0 = \varepsilon_m z_0$; $\varepsilon_m = \max \varepsilon(\mu)$.

We consider the scattering characteristics of the film in ON and OFF states. We neglect by the cooperative effects arising at dense packing of droplets and believe that there is no absorption in liquid crystal droplets, but the matrix itself can absorb light owing to a dye dissolved in the matrix.

In the OFF state, the scattering and absorbing properties of the medium are determined by the phase function $X^-(\cos\gamma)$, the scattering coefficient σ^-, and the absorption coefficient α. These quantities are related to the scattering characteristics of separate droplets in the following way

$$\sigma^- = w\sigma_0^- = w\frac{\overline{\Sigma}_s}{v}, \quad (3)$$

$$\alpha = (1 - w)\alpha_0, \quad (4)$$

$$X^-(\theta) = \frac{1}{\overline{\Sigma}_s}\frac{d\overline{\Sigma}_s}{d\Omega}(\theta), \quad (5)$$

where v is the mean volume of liquid crystal droplets; w is the volume concentration of droplets; $\overline{\Sigma}_s$ is the scattering cross section averaged over the sizes and directions of droplet directors; $\sigma_0^- = \frac{\overline{\Sigma}_s}{v}$ is the scattering coefficient of liquid crystal droplets in the OFF state at unit volume concentration; $\frac{d\overline{\Sigma}_s}{d\Omega}(\theta)$ is the differential cross section of droplets averaged over the sizes and directions of droplet directors; α_0 is the absorption coefficient of the polymer.

In the ON state, the medium is characterized by the azimuth-averaged phase function $X^+(\mu,\mu')$, the scattering coefficient σ^+, and the absorption coefficient α.

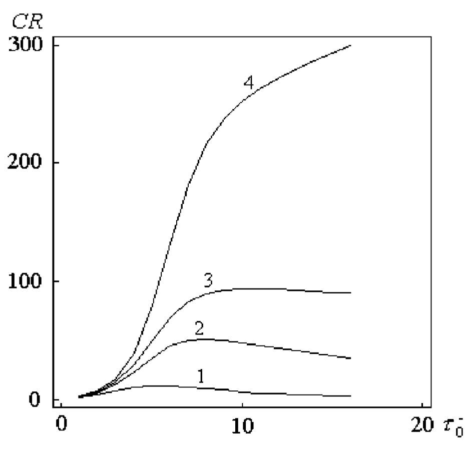

FIGURE 3. Dependence of *CR* on optical thickness τ_0^- (in the OFF state) at $\overline{R}$=0.2 ìm, C_v=0.4. n_e=1.7, $n_0 = n_m$=1.55 at $\Delta\theta_c = 1^\circ$. $\theta_{0e} = \theta_e = 22.4^\circ$ (curve 1), 16.4° (2), 13.2° (3), 6.8° (4).

OPTICAL PARAMETERS OF A UNIT VOLUME

To determine the unit volume parameters,[7, 9, 10] we used the anomalous diffraction approach (ADA). The angular distribution of scattered light in this approach is well described in the region of small angles ($\gamma \leq 30^o$) and basically cannot be calculated at $\gamma > 90^o$. To give the angular distribution of scattered light in the region of large angles, we extrapolated the size-averaged differential cross section of scattering, found in the ADA, by the exponential dependence. We choose the log-normal size-distribution of droplets, which is often used for PDLC droplets polydispersity description.[15, 16]

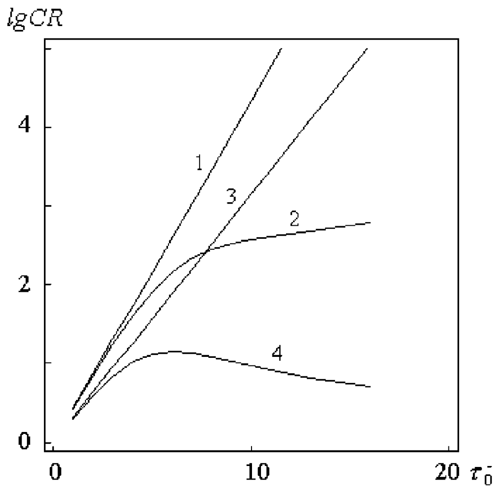

FIGURE 4. Dependencies of *lg CR* on the optical thickness τ_0^- without (curves 1, 3) and with regard to of multiply scattered light (curves 2, 4) at $\overline{R}$=0.2 ìm, C_v=0.4, n_e=1.7, $n_0 = n_m$=1.55, $\Delta\theta_c = 1^\circ$, $\theta_{0e} = 6.81^\circ$ (curves 1, 2), $\theta_{0e} = 22.4^\circ$ (curves 3, 4).

Figure 2 gives the typical angular dependencies of the scattering coefficient for nonabsorbing liquid crystal droplets in ON and OFF states. Hereinafter the calculations are carried out at $\lambda = 0.5\mu m$. For PDLC films in the ON state when the droplet directors are oriented perpendicular to the interface, the scattering coefficient has a characteristic minimum whose position depends on the ratio between the refractive indices of the liquid crystal and the refractive index of the matrix.[12, 15, 17, 18] In the OFF state, the value of the scattering coefficient does not depend on light propagation direction.

LIGHT INTENSITY AT THE AIR-MATRIX INTERFACES

Let us consider a homogeneous layer of a light-scattering and absorbing disperse medium. If the Fresnel reflection at the interfaces is absent, the radiance factors of reflected $\rho(\mu,\mu')$ and transmitted $\sigma(\mu,\mu')$ light are determined by the equations:[7, 9, 10]

$$I^{\downarrow}(z=0,\mu) = \int_0^1 2\rho(\mu,\mu')\mu' I_0(\mu')d\mu' , \tag{6}$$

$$I^{\uparrow}(z=z_0,\mu) = e^{-\tau_0 \frac{e(\mu)}{\mu}} I_0(\mu) + \int_0^1 2\sigma(\mu,\mu')\mu' I_0(\mu')d\mu' . \tag{7}$$

Here $I_0(\mu)$ is the intensity of light incident on the film. $I^{\downarrow}(z=0,\mu)$ the back-scattered light intensity at the upper interface; $I^{\uparrow}(z=z_0,\mu)$ the forward-scattered light intensity at the lower interface.

To determine the radiance factors $\rho(\mu,\mu')$ and $\sigma(\mu,\mu')$ of an anisotropic medium, the calculation technique based on the layer doubling method has been used. The modification of the method for media with characteristics independent of light propagation direction is described elsewhere.[10] We used the version of the method which takes the particle anisotropy into account.[19]

CONTRAST RATIO

Contrast ratio (*CR*) depends on film properties, illumination and observation conditions.

Initially we consider *CR* of a film illuminated by an azimuthally symmetrical conical wide light beam with vertex angle $\theta_e = \theta_{0e}$ of the cone (see Figure 1). A receiver collects transmitted light under the angle θ_{0e}.

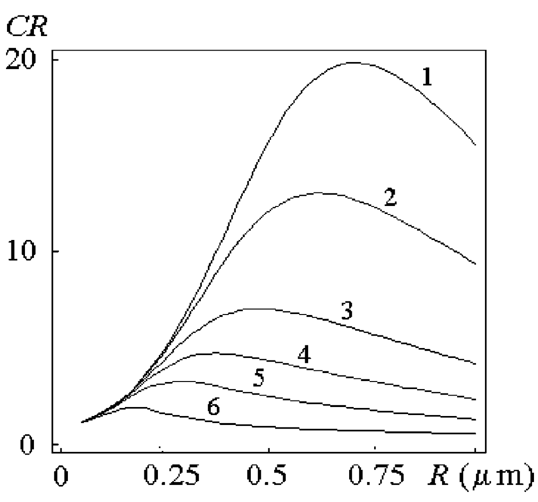

FIGURE 5. Dependence of *CR* on droplet radius at normal illumination by parallel beam for collection angle θ_c=2° (curve 1); 3° (2); 5° (3); 7° (4); 10° (5); 20° (6). n_e=1.7, n_0=n_m=1.55, wz_0=10.

Figures 3 and 4 show calculated dependencies of the contrast ratio. To gain a more penetrating insight into the influence of contributions of scattered and directly passed light on the contrast ratio formation, we present the *CR* values calculated with account for only directly passed light. The corresponding results are given in Figure 4. As seen from this figure, the main contribution to the contrast ratio at small optical thicknesses comes from the directly passed light. In the region of thicknesses where pairs of curves (1 and 2, 3 and 4) are close to each other, it is possible to restrict ourselves by the single-scattering approximation for the CR estimation. As it is obvious, the account for scattered light leads to a decrease in the CR. The qualitative estimation of the scattered light influence for thin and thick films as well as the collection angle influence can be made for any practical situation using the above equations.

At normal illumination the dependence of contrast ratio *CR* on the radius at constant volume of droplets in the film is shown in Figure 5. It illustrates the situation when the receiver fully collects directly transmitted light and a part of scattered light. The latter is determined by the collection angle θ_c. With the increasing collection angle, the position R_m of the maximum on the dependence *CR*-*R* shifts from $R_m \approx 0{,}75\mu m$ to $R_m \approx 0{,}2\mu m$. The shift of R_m to the smaller size values is due to an increasing contribution of multiply scattered light with the increasing collection angle.

CONCLUSION

The model for the calculation of the radiative transfer in PDLC films with regard to the anisotropy of nematic liquid crystal droplets, the polydispersity of droplets, multiple scattering, and Fresnel reflection at the film interfaces has been developed. The model allows one to analyze the angular characteristics of reflected and transmitted light, contrast ratio as a function of illumination conditions, collection angles, spectral range of incident light, size and concentration of droplets, applied field, and director field configuration in the droplet. Some of the results can be used for the consideration of another types of composite LC films,[20,21] where the multiple light scattering regime is achieved.

Acknowledgments

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